



Correcting GOES-R Magnetometer Data for Stray Fields

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Delano Carter, Douglas Freesland, Sivakumar Tadikonda, Jeffrey Kronenwetter, Monica Todirita,

Melissa Dahya, Donald Chu

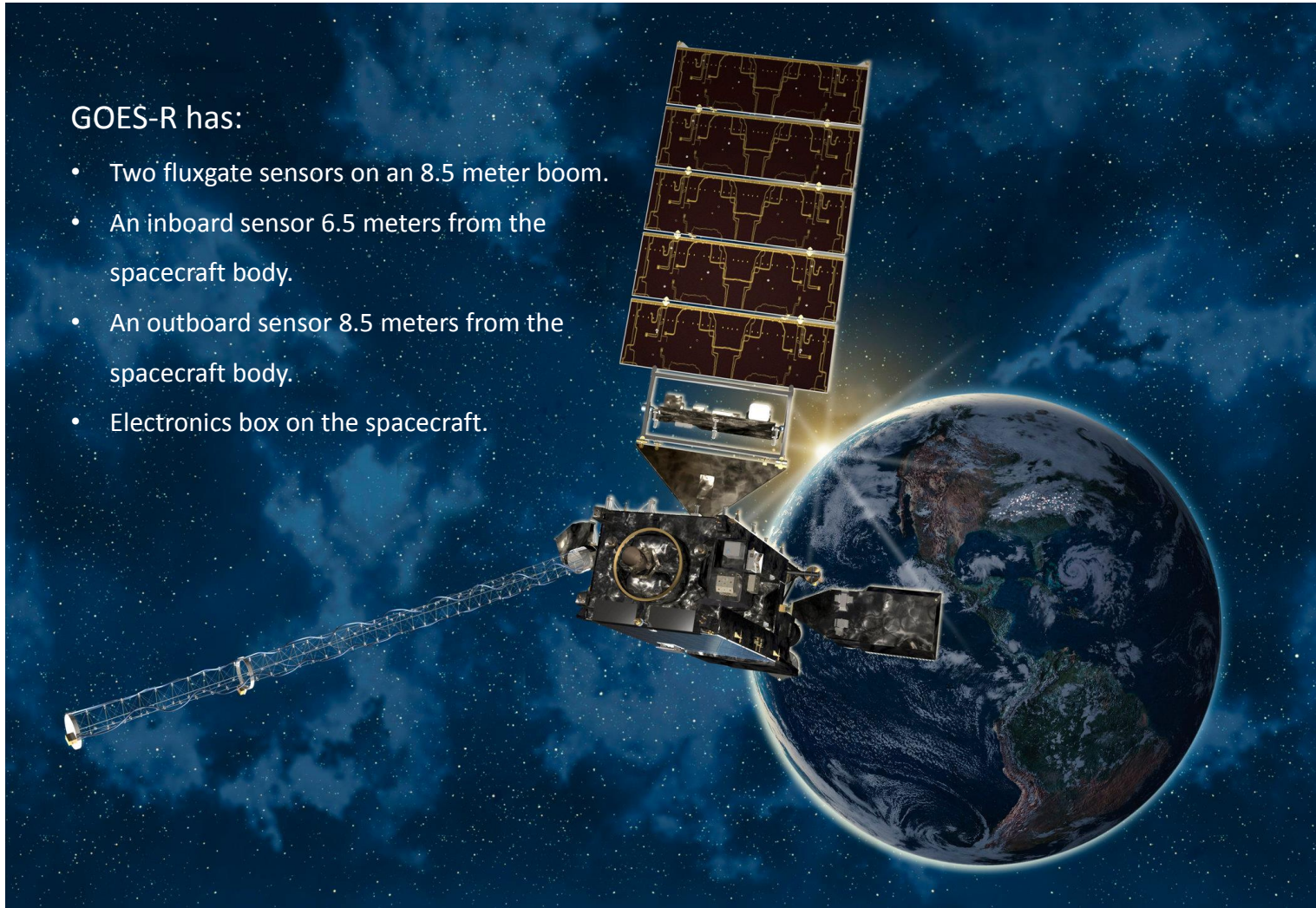
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GOES-R Magnetometer Subsystem



GOES-R has:

- Two fluxgate sensors on an 8.5 meter boom.
- An inboard sensor 6.5 meters from the spacecraft body.
- An outboard sensor 8.5 meters from the spacecraft body.
- Electronics box on the spacecraft.





Average Errors

- Absolute mean plus 3 sigma error < 1.7 nT
- Computed over a day
- Quiet day field

Worst Case Errors

- Transients > 0.3 nT
- Fewer than one per hour
- None more than 5 seconds in duration



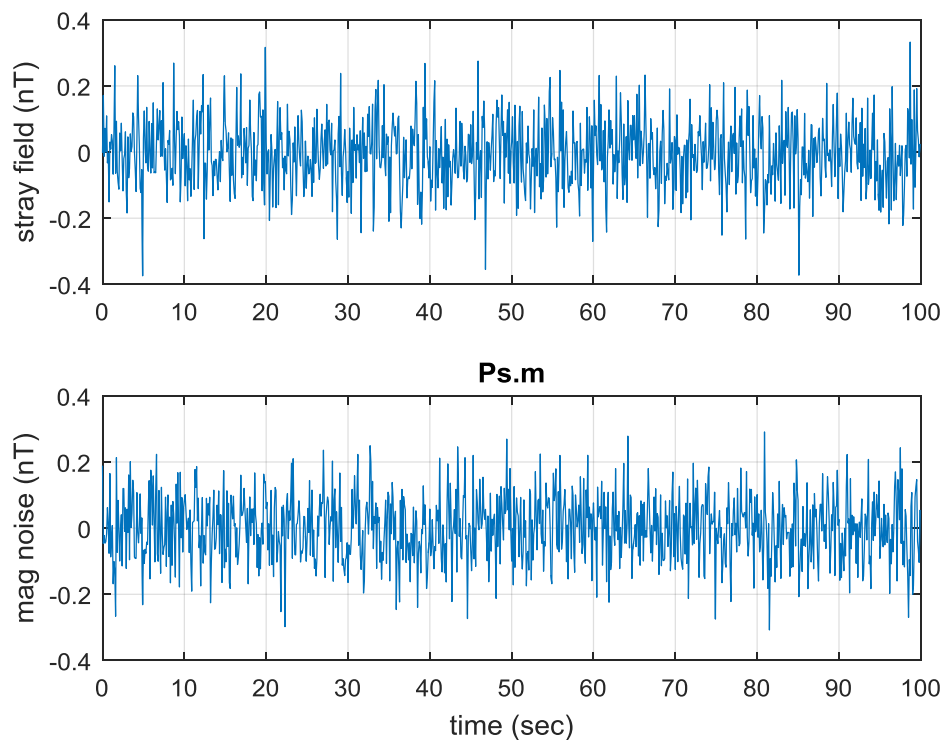
With two magnetometers on a boom, we can average and estimate the ambient field only, or we can solve for both ambient and spacecraft fields. The second approach is called the gradiometer algorithm, and it provides a simple way to remove spacecraft stray fields.

- The problems with the gradiometer algorithm are:
 1. the dipole has to be along the boom axis
 2. stray field dipole location must be known
 3. solving for stray fields as well as ambient fields adds noise
- In this analysis, we:
 1. ignore the first two problems
 2. assume simple noise and stray field models
 3. compare the benefit of solving for stray field with the detriment of adding noise
 4. examine other ways of removing stray fields

Noise and Stray Field Models

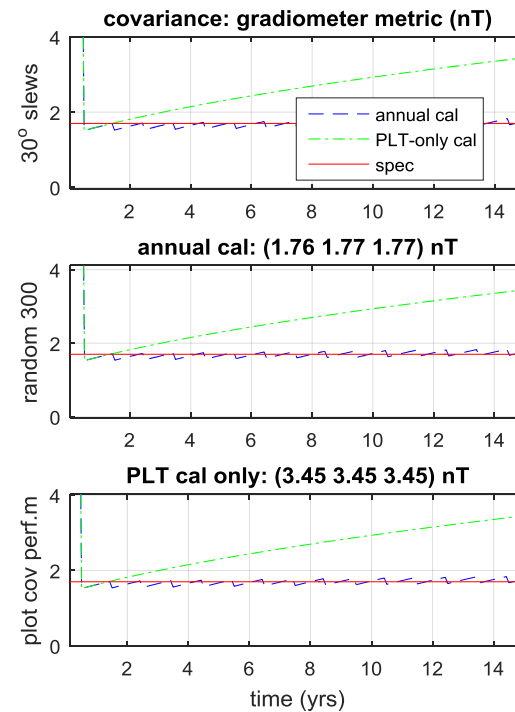
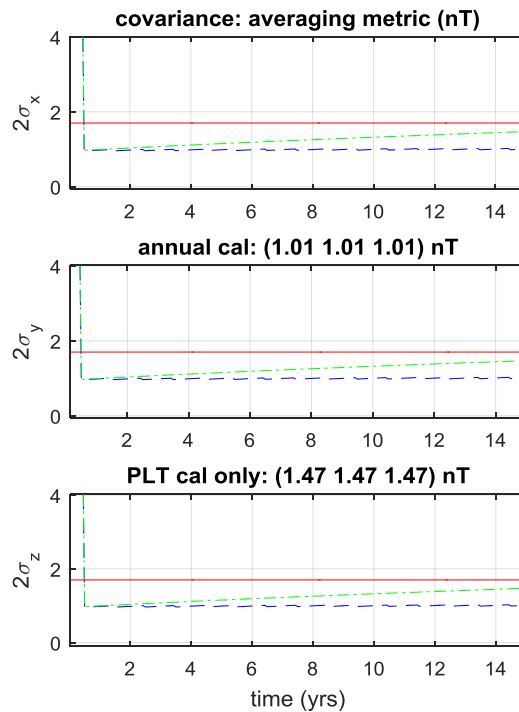


- We model both stray fields and magnetometer noise as isotropic zero-mean Gaussian white noise.
- Stray field variance σ_s^2 is used to compute the averaging consider covariance.
- Noise variance σ^2 is used to predict noise covariance for both averaging and the gradiometer.



Averaging or Gradiometry?

To help decide the question, we simulated the two algorithms. For our assumed noise and stray field levels (both around 0.1 nT), averaging gave about half the error of the gradiometer.



- We want the ambient magnetic field \vec{B}^A , but the inboard \vec{B}^{IB} and outboard \vec{B}^{OB} magnetometer measurements are corrupted stray fields \vec{B}^S and noise \vec{n}

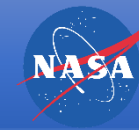
$$\vec{B}^{IB} = \vec{B}^A + \vec{B}^S(\vec{r}^{IB}) + \vec{n}^{IB}$$

$$\vec{B}^{OB} = \vec{B}^A + \vec{B}^S(\vec{r}^{OB}) + \vec{n}^{OB}$$

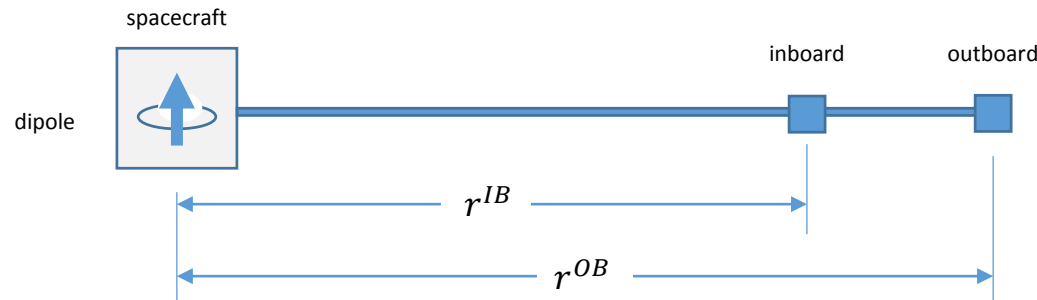
- Stray field may be approximated by that of a magnetic dipole \vec{m} . If the dipole-to-magnetometer vector is \vec{r} , the stray field is

$$\vec{B}^S = \frac{\mu_0}{4\pi r^3} (3\hat{r}\hat{r}^T - I_3)\vec{m} = \beta(\vec{r})\vec{m}$$

Gradiometry or Averaging?



- Is it better to solve for the stray field as the gradiometer does or just average and reduce noise?
- We can compute the noise covariance of the algorithms, but the answer depends on the stray field size.
- For that, we add a consider covariance to the averaging noise covariance.
- If the gradiometer variance is still larger than the total averaging covariance, averaging is preferable.



The gradiometer model assumes the stray field dipole and magnetometer positions are collinear and that their separations are known. No attempt is made here to determine position sensitivity. We define outboard-to-inboard separation ratio to be ρ

$$\rho = r^{OB} / r^{IB} \geq 1$$

- Dipole strength falls off as the third power of distance, and direction does not change with distance

$$\vec{y} = \begin{pmatrix} \vec{B}^{IB} \\ \vec{B}^{OB} \end{pmatrix} = \begin{pmatrix} I_3 & I_3 \\ I_3 & I_3/\rho^3 \end{pmatrix} \begin{pmatrix} \vec{B}^A \\ \vec{B}^S \end{pmatrix} = H\vec{x}$$

- If σ^2 is the observation noise variance, the normal matrix is

$$H^TWH = \frac{1}{\sigma^2} \begin{pmatrix} 2I_3 & (1 + \rho^{-3})I_3 \\ (1 + \rho^{-3})I_3 & (1 + \rho^{-6})I_3 \end{pmatrix}$$

- The inverse of H^TWH is the state covariance matrix P

$$P = \frac{\sigma^2}{(\rho^3 - 1)^2} \begin{pmatrix} (1 + \rho^6)I_3 & -\rho^3(1 + \rho^3)I_3 \\ -\rho^3(1 + \rho^3)I_3 & 2\rho^6I_3 \end{pmatrix}$$

- This gives the gradiometer ambient field variance σ_G^2 as (upper left corner entry)

$$\sigma_G^2 = \frac{1 + \rho^6}{(\rho^3 - 1)^2} \sigma^2$$



Accuracy

- Absolute mean plus 3-sigma error < 1.7 nT in quiet fields
- Absolute mean plus 2-sigma error < 1.7 nT in storm fields
- Computed for worst case *day* in 15-year mission*

Transients

- Greater than 0.3 nT
- Fewer than one per hour
- Lasting no more than 5 seconds

* Time span over which to compute the accuracy metric was left out of the requirements.

- If we solve for \vec{B}^A as the average of the two magnetometer measurements, the observation model is

$$\vec{y} = \begin{pmatrix} \vec{B}^{IB} \\ \vec{B}^{OB} \end{pmatrix} = \begin{pmatrix} I_3 \\ I_3 \end{pmatrix} \vec{B}^A = H_A \vec{x}$$

- The noise covariance P_n is $\sigma^2 I_3 / 2$. This is not total covariance because it ignores error due to stray field. To account for this, we add a “consider covariance” P_c . If P_S is the stray field covariance $\sigma_S^2 I_3$, its contribution is

$$P_c = T P_S T^T = (P_n H_A^T W H_S) P_S (P_n H_A^T W H_S)^T$$

- The total covariance P_A is the sum of P_n and P_c . The averaging ambient field variance σ_A^2 is then

$$\sigma_A^2 = \frac{\sigma^2}{2} + \left(\frac{\rho^3 + 1}{2\rho^3} \right)^2 \sigma_S^2$$

- If gradiometer variance σ_G^2 is greater than averaging variance σ_A^2 , averaging is the better choice. The variance ratio is

$$\frac{\sigma_A^2}{\sigma_G^2} = \frac{(\rho^3 - 1)^2}{1 + \rho^6} \left(\frac{1}{2} + \left(\frac{\rho^3 + 1}{2\rho^3} \right)^2 \frac{\sigma_S^2}{\sigma^2} \right)$$

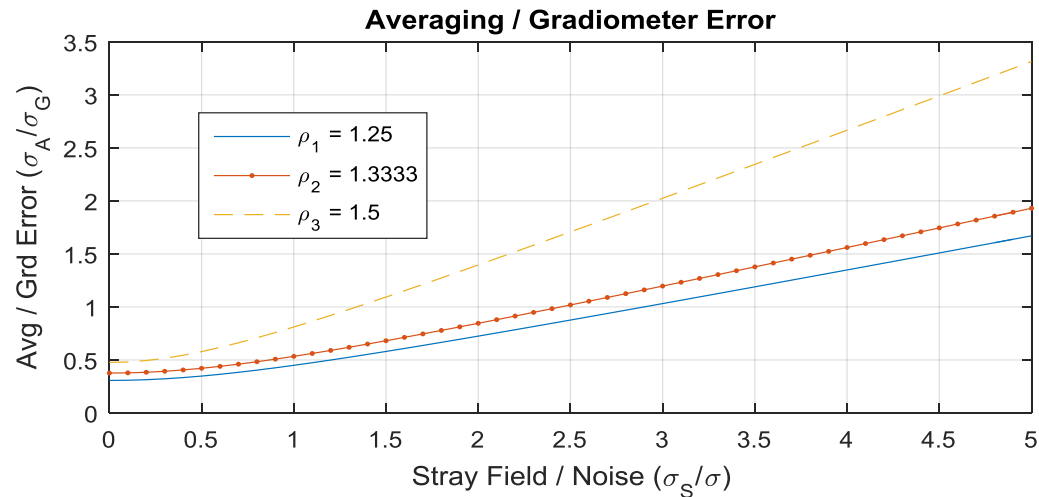
- Equating variances gives the stray field variance beyond which the gradiometer becomes the better choice

$$\frac{1 + \rho^6}{(\rho^3 - 1)^2} \sigma^2 = \frac{\sigma^2}{2} + \left(\frac{\rho^3 + 1}{2\rho^3} \right)^2 \sigma_S^2$$

- Solving for the transition ratio gives

$$\frac{\sigma_S}{\sigma} = \sqrt{2} \frac{\rho^3}{\rho^3 - 1}$$

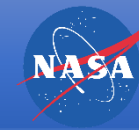
- For the GOES-R outboard-to-inboard distance ratio of 1.33, averaging is preferable to the gradiometer unless stray field standard deviation is 2.5 times that of the noise. We expect both noise and stray field standard deviations to be 0.1 nT.



For GOES-R, ρ is roughly 1.3, so the stray field has to be about 2.5 times as large as the noise for the gradiometer to be better than averaging.



1. Perform a running average on the gradiometer to reduce the noise.
 - The problem with the gradiometer is its amplification of the magnetometer noise.
 - This is probably the easiest and most realistic option.
2. Eliminate averaging spikes not in the gradiometer solutions.
 - This requires recognizing spikes and checking against the gradiometer solutions.
 - It would also require running both algorithms in parallel.
3. Preprocess the raw magnetometer data to remove known stray fields.
 - This requires working with the level 0 data.
 - It is the most ambitious but probably the most accurate of the three approaches.



- Because we believe that we can calibrate for static spacecraft fields, our primary concern now is with time-varying, *i.e.* stray fields.*
- Doug Westbury of Lockheed Martin measured every spacecraft assembly for compliance with the magnetic control requirements.
- The three that were found to generate the largest stray fields were the solar array, the arcjet thrusters and the reaction wheels.

* *There is still the possibility that the spacecraft dipole moment could reduce the useful range of magnetometer measurements.*

- The solar array is divided into 16 circuits. To reduce solar array field, half are wound clockwise and half counterclockwise. The magnitude of the net dipole moment m^{SA} is the sum of the effective solar array circuit areas A_i^{SA} times the circuit currents i_i^{SA}

$$m^{SA} = \sum_{i=1}^n m_i^{SA} = \sum_{i=1}^n A_i^{SA} i_i^{SA}$$

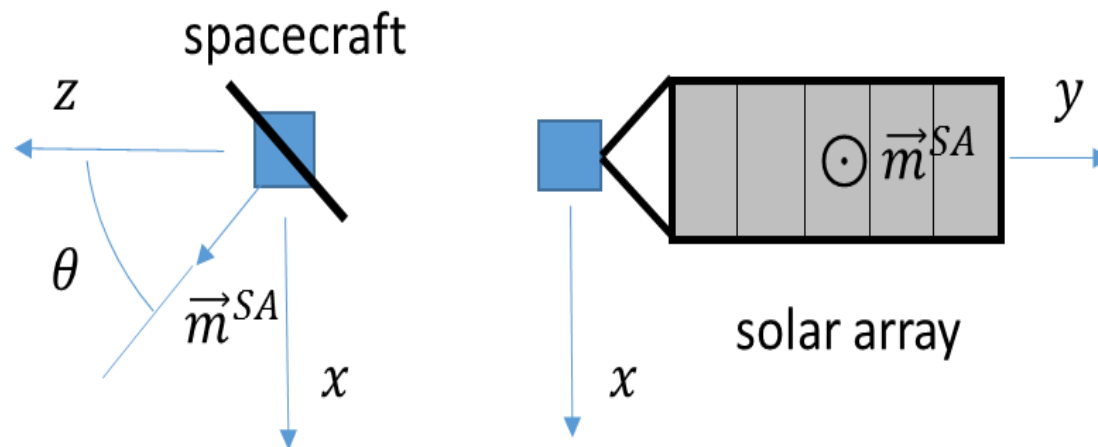
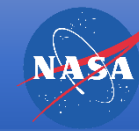
- The dipole moment direction is perpendicular to the solar array and is a function of the solar array drive angle θ

$$\vec{m}^{SA} = m^{SA} \begin{pmatrix} \sin\theta \\ 0 \\ \cos\theta \end{pmatrix}$$

- If \vec{r}^{SA} is the solar array dipole-to-magnetometer vector, the solar array field is

$$\vec{B}^{SA} = \beta(\vec{r}^{SA})\vec{m}^{SA}$$

Solar Array Drive Angle θ



The net solar array magnetic dipole is the sum of the individual circuit dipole moments and is located at the center of the solar array.

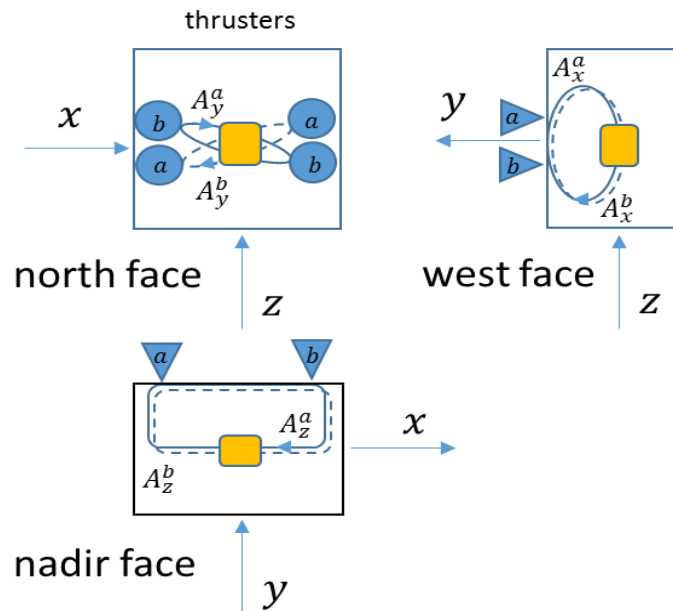
- For orbital inclination control, there are four arcjets on the $-y$ (north) face of the spacecraft fired two at a time. Although they may not themselves generate much field, the electrical current they require does.
- The two arcjet-pair dipole moments $\vec{m}^{a/b}$ are a function of the currents $i^{a/b}$ and the circuit areas $A_i^{a/b}$ normal to the x, y and z spacecraft axes. These three areas may be written as a vector $\vec{A}^{a/b}$ such that the two arcjet dipoles take the form

$$\vec{m}^{a/b} = \begin{pmatrix} A_x^{a/b} & A_y^{a/b} & A_z^{a/b} \end{pmatrix}^T i^{a/b} = \vec{A}^{a/b} i^{a/b}$$

- If \vec{r}^{AJ} is the arcjets-to-magnetometer vector, the arcjet fields can then be written as a linear function of the arcjet current

$$\vec{B}^{a/b} = \beta(\vec{r}^{AJ}) \vec{A}^{a/b} i^{a/b}$$

Arcjet Dipole Moment Areas $A^{a/b}$



We want to estimate the projection of the circuit area onto the xy , yz and zx planes. The component of the dipole moment perpendicular to the planes is then the current times these areas.

- Rotor moment for wheel i may be expressed in terms of unit basis vectors \hat{u}_i and \hat{v}_i fixed in the wheel frame. Predicting fields requires wheel speeds ω_i^{RW} , rotor phases ψ_i^{RW} and the rotor dipole moments m_i^{RW} . We can sum the moments as

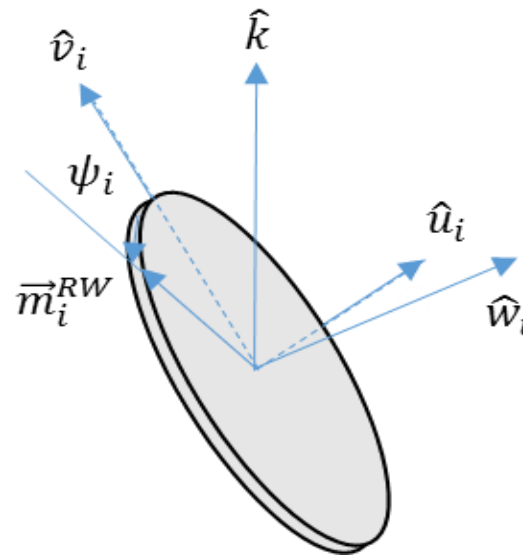
$$\vec{m}^{RW} = \sum_{i=1}^6 m_i^{RW} (\cos\psi_i \hat{u}_i + \sin\psi_i \hat{v}_i)$$

- If \hat{k} is the unit vector along the spacecraft z-axis, the \hat{u}_i and \hat{v}_i unit vectors are

$$\hat{u}_i = \hat{k} \times \hat{w}_i / |\hat{k} \times \hat{w}_i| \quad \hat{v}_i = \hat{w}_i \times \hat{u}_i / |\hat{w}_i \times \hat{u}_i|$$

- If \vec{r}^{RW} is the wheels-to-magnetometer vector, compute wheel dipole moment vector \vec{m}^{RW} and the field it produces as

$$\vec{B}^{RW} = \beta(\vec{r}^{RW}) \vec{m}^{RW}$$



We measure rotor phase ψ as the angle of the dipole moment vector \vec{m}^{RW} from the wheel assembly \hat{u} and \hat{v} axes.

To remove the solar array, arcjet and reaction wheel fields as outlined above, we need to know:

1. Solar array drive angle, circuit areas and currents
2. Arcjet effective circuit areas, normal vectors and currents
3. Rotor dipole moments, rotation rates and phases

- The solar array drive angle and currents come in telemetry, so the important remaining items to determine are the circuit areas. On the ground, known currents were forced through the solar array circuits, and the resulting magnetic fields were measured at two heights above the array. These were then normalized by the applied currents i_i^{SA} to give the effective areas A_i^{SA} for each circuit

$$A_i^{SA} = m_i^{SA} / i_i^{SA}$$

- On-orbit we hope to dither the solar array and estimate its dipole moment from the resulting field variation. We have to choose dither amplitude and frequency that satisfy the solar array angular velocity ω and acceleration α constraints while changing the field appreciably at a frequency well above that of ambient variations.

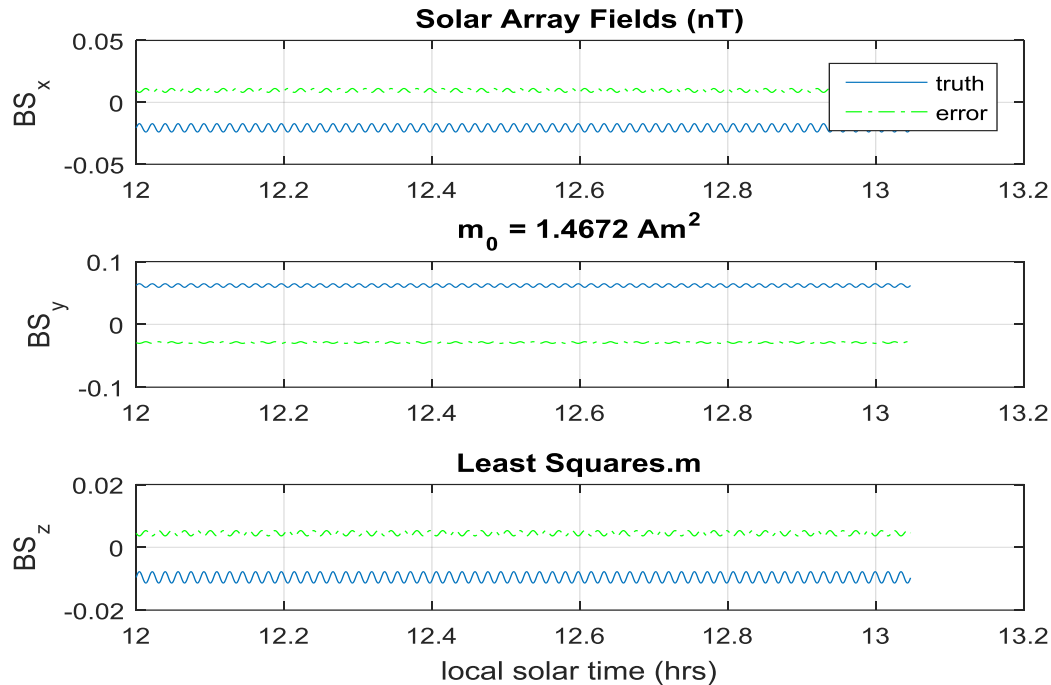
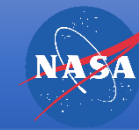
- Maximum solar array angular velocity ω and acceleration α constrain Ω and Θ are

$$\omega = \dot{\theta} \leq \Omega \Theta \leq \omega_{max}$$

$$\alpha = \ddot{\theta} \leq \Omega^2 \Theta \leq \alpha_{max}$$

- Dithering causes the solar array field to vary about a non-zero mean value. To estimate the dipole moment, we first subtract the average field value from both the observation and the prediction. A direct search then minimizes the sum of squared errors.
- With one hour of 5° dithering, we expect to estimate the solar array dipole moment to 0.5 Am^2 (1σ) accuracy. Although less accurate than ground measurement, it does provide a check under flight-like conditions. If we were to use these in-flight estimates, we would still reduce solar array stray fields by half.

Reduction of Solar Array Fields



Estimating the solar array dipole moment reduces the uncertainty to 0.5 Am^2 and cuts the expected solar array field error in half.

- It is not possible to fire the arcjets on the ground with the cabling in a flight-like configuration, so arcjet magnetic characterization has to be on-orbit. In normal operations, the arcjets are fired every few days in one long pulse. The on- and off-transitions should be quite sharp, and we know when to look for them.
- As above, the procedure would be to solve a least squares problem for the arcjet dipole moment using the change in the observations $\Delta \vec{B}^{IB/OB}$ before and after the transitions

$$\begin{pmatrix} \Delta \vec{B}^{IB} \\ \Delta \vec{B}^{OB} \end{pmatrix} = \begin{pmatrix} \beta(\vec{r}^{IA}) \\ \beta(\vec{r}^{OA}) \end{pmatrix} \vec{m}^{a/b}$$

where \vec{r}^{IA} and \vec{r}^{OA} are the arcjet-to-inboard and -outboard magnetometer position vectors.

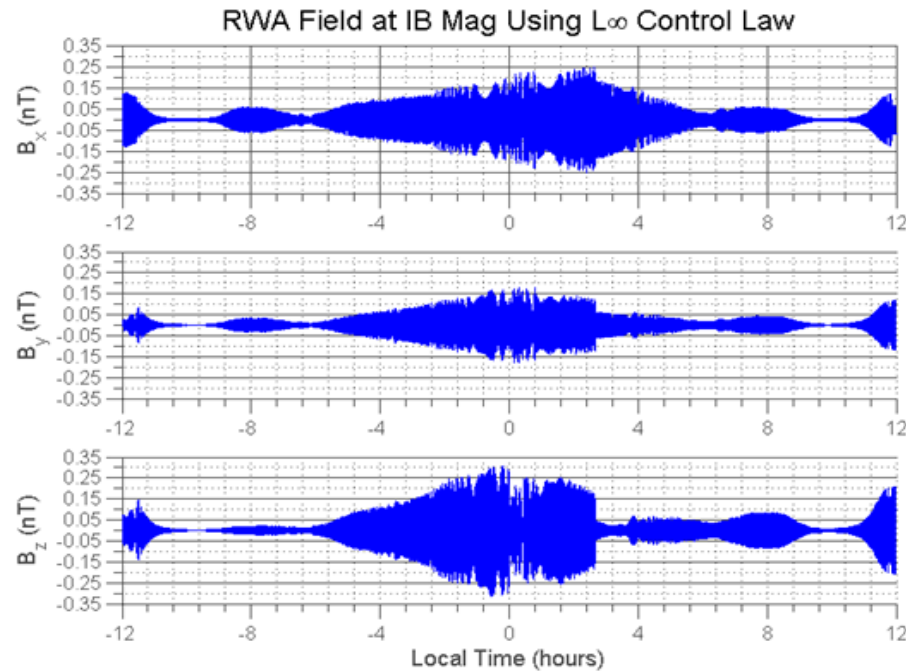


- Without noise, it would only take one a and one b arcjet pair pulse to determine the fields.
- The magnetometer noise σ plus the ambient field variability σ_B , however, make it necessary to average multiple firings.
- If over the time it takes the magnetometer to respond to the arcjet step transition, the ambient field does not change, the only noise is from the magnetometer itself.
- In this case, it would take 100 transitions (50 pulses) to reduce the 0.10 nT magnetometer noise to the 0.01 nT level we might want for arcjet field knowledge.



- Before assembly, Doug Westbury measured the magnetic dipole moment of each of the six reaction wheel rotors. Assuming the dipole moment does not change, if we knew the rotor orientations, *i.e.* phase angles, we could predict the reaction wheel fields at the magnetometers.
- Unfortunately, there is no rotor phase telemetry, so we have to estimate phase. We use wheel speeds to propagate phase between observations, *i.e.* times when the rates are within the magnetometer passband. During those times, we estimate the rotor fields and remove them from the magnetometer readings.
- When wheel speeds are within the magnetometer passband, measurements are corrupted. Wheel speeds are used as inputs to a bank of Least Mean Squares (LMS) based adaptive filters. Because all six wheels may be within the measurement passband, the required number of filters is six, *i.e.* one per wheel.

Typical Reaction Wheel Fields



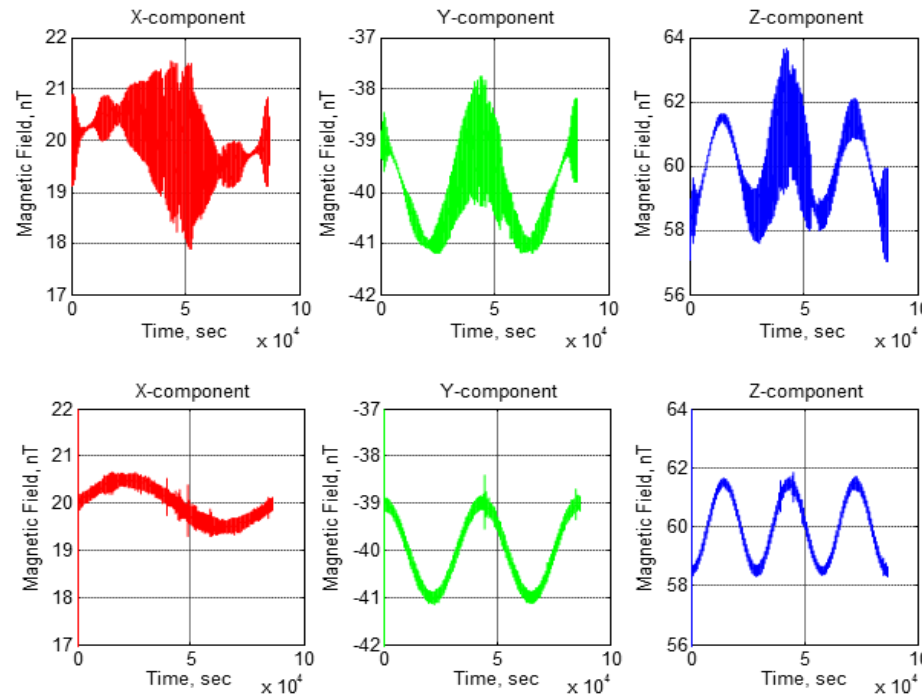
This is actually a worst case in which all six reaction wheel speeds are in-band. The magnitude was also multiplied by a factor of seven to make it more noticeable.



Each LMS adaptive filter is responsible for estimating the wheel dipole phase angle and amplitude.

- There are two inputs to each LMS adaptive filter. One is the tach signal, and the other is the filter effectivity error. Filter effectivity error is a measure of how well the filter is removing the undesired wheel field. Based on these two inputs, the LMS adaptive filter generates a correction signal to remove the dipole field.
- To prevent the LMS filters from competing with each other in a detrimental manner, the learning rates for the six LMS adaptive filters are skewed. This has the effect of permitting some filters to converge to wheel magnetic dipole signals quicker than other filters.
- The top row shows the simulated reaction wheel fields superimposed on a sinusoidally-varying ambient field. The traces are wide when the wheel speeds go through the magnetometer passband. The bottom row shows the magnetometer readings after being corrected with the LMS filter estimates.

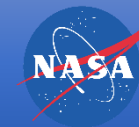
Reduction in Reaction Wheel Effects



The slow diurnal sinusoid is the assumed ambient field. The thick traces at the top are corrupted by the exaggerated reaction wheel fields. The thin traces have had the wheel fields removed by the LMS filters.

There are multiple ways of dealing with stray magnetic field errors in space-based magnetometer systems.

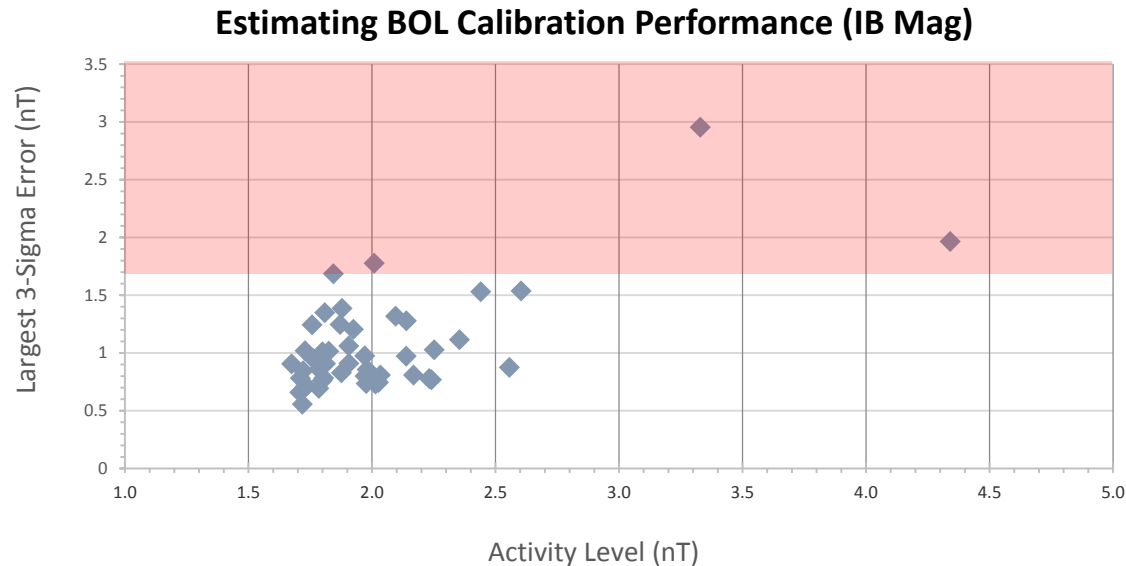
- We have examined gradiometer noise sensitivity and recommended when to use gradiometry and when to average. Unless stray fields are twice as large as magnetometer noise, covariance analysis suggests that it is preferable for GOES-R to average rather than use the gradiometer.
- We have also outlined models for three common sources of stray fields, *i.e.* solar arrays, arcjets and reaction wheels, and suggested how the necessary parameters can be measured and the stray fields removed. It may be prudent to add these steps to the ground processing.
- One question not covered is on-orbit performance verification. How will we know that any corrections we make actually help? There may be times when we are collocated with other GOES satellites, but most of the time there will be no reference nearby. This and other operations questions remain to be addressed.



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- Finally, the authors thank intern Elizabeth Singleton for her work on solar array characterization.



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When the magnitude of the field change over the calibration period exceeds 3 nT, the bias error was found to be significantly more uncertain.